The optimal inheritance tax in the presence of Investment in Education

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November 2012

This paper calculates the optimal inheritance tax in a model in which inheritances are used to finance investment in education. Two results are obtained: 1) The optimal inheritance tax schedule includes a threshold, estimated between 2.5 and 5.5 times per-capita gdp. This result holds for a Rawlsian Social planner that maximizes the welfare of the poorest individual, who does not leave bequests. 2) Contrary to the result of a 100 percent tax on accidental bequests, the optimal simulated tax rates are between 28 to 42 percent. These results are in line with existing schedules in developed economies.

Key Words: *Investment in Education, Inheritance Tax, Threshold.*

JEL Classification Nmbers: H21, H23

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1. **Introduction**

About half of developed and developing economies apply an inheritance tax (Table 1). Developed economies, among them the U.S., impose a tax with a threshold, avoiding taxes on inheritances and gifts under a certain amount, which in the US will be 1 million dollars starting in 2013(Table 2).

It is well-known that one of the reasons for not imposing a gift/inheritance tax by so many countries is the difficulty of monitoring gifts/inheritances, which makes this tax hardly implementable.[2] However, this reason seems convincing in developing countries, in which taxation is mostly indirect and the information on income and gifts/inheritances is scarce. In developed economies, where a compulsory income/estate declaration exists and the available information is of good quality, it is odd to mention this reason to explain the lack of implementation of an inheritance tax in about half of them.[3]

Another aspect related to the low level of implementation of this tax is the lack of consensus among policy-makers on the optimal tax schedule that should be imposed. According to existing literature, the optimal inheritance tax schedule should not have a threshold ; for egoistic individuals, leaving unintended bequests, the tax rate should be 100 percent, while for altruistic individuals the optimal tax rate should be close to 60 percent (Blumkin and Sadka, 2003). High inheritance tax rates without any threshold is a result that does not match the policy pursued by policy-makers in real life, and it calls for new models. The main purpose of this paper is extending the model to the case in which individuals care about the future generation, by financing its investment in education,[4] and check whether in

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[2] Gales, Hines and Slemrod (2001) shed light on different aspects of the inheritance tax.

[3] Note that both developed and developing economies shall accept a certain degree of tax avoidance. Tax voidance is a natural reaction to taxes in general, and to inheritance taxes in particular. Graetz and Shapiro (2005) show that successors tend to quickly sell properties as a way of dealing with the inheritance tax.

[4] A recent survey by Hart Research Associates in swing states toward 2012 elections found that for 67% of respondants education will be extremely important for them personally in this year's elections for President and Congress; 90% of voters feel it is extremely (69%) or fairly (21%) important for their governor and state legislature to adress the issue of education as a matter of state policy; 82% would most likely vote for a candidate that promotes allowing employers to offer tuition assistance to employees tax free.

the new framework it is optimal to impose a threshold, and whether simulated optimal tax rates are in the range implemented by policy makers. While educational expenditures by parents in real life are not exempted through the gift and inheritance tax, the present framework can be seen as a benchmark for analyzing the optimal threshold under the assumption that benevolent governments are willing to exempt intergenerational transfers used for educational expenditures. In fact, tax provisions in different countries provide evidence for such a willingness among governments (see section 3.1).

The first paper providing a rationale for a threshold in the optimal inheritance tax schedule is Farhi and Werning (2010), who justify, using a model of altruistic agents, a progressive inheritance tax schedule for estates\inheritances. If the social planner utility function includes the welfare of the future generation, then it is optimal to impose a progressive inheritance tax with a subsidy for bequests by low income dynasties, and a progressive tax on inheritances of high income dynasties. By acting this way, the consumption of the rich (poor) at the present generation becomes more (less) attractive than bequests, and thus the transmission of inequality to the next generation is softened, providing an optimal outcome from the point of view of the social planner. If bequest subsidies are not allowed (for example because of problems of implementability), then the optimal schedule is to impose a threshold until a certain level of bequests, and since that level onwards imposing a progressive inheritance tax schedule. However, note that in this paper the result about the optimality of the threshold is obtained in an ad-hoc manner, by imposing the non-negativivity constraint on taxes. Saez and Piketty (2012) also calculate optimal inheritance taxes for altruistic individuals, and find that for realistic parameters the tax rate should be in the range between 50 to 60 percent. In one of their sub-sections they find that the optimal inheritance tax shall be non-linear; however, they do not find a justification for a threshold which is adopted by assumption. As opposed to these papers, a novel aspect of the present paper, is that the threshold is obtained as a solution of the model.

Moreover, in my model altruistic bequests coexist with accidental bequests.[5]

The present paper provides a different rationale for the actual policy implemented by most countries imposing an inheritance tax, in a model in which gifts and bequests are used for investing in education of the future generation. Since young people do not have their own funds, bequests and gifts constitute a basic source for financing education. In fact, in terms of the life cycle model, the newborn generation finances his\her education at a stage in which he\she doesn't have his own economic resources, and thus he\she must get aid from his\her family, or otherwise obtain a loan from financial institutions. Galor and Zeira (1993) show that the lack of access for young and talented individuals to financial sources is one of the market failures that do not allow them to invest in education, causing income inequality to be transmited among generations, which will be called in this paper "dynasties". These dynasties invest in education by transferring resources from parents to children through gifts/bequests, that are used by the newborn to acquire human capital.[6] The novel aspect of this paper is to characterize the optimal inheritance tax given this market failure. As in Saez and Piketty (2012), this task will be performed in a world where both income and inheritance taxes exist, since both of them are separate sources for inequality.

In particular, it is interesting to characterize the optimal schedule for a Rawlsian social planner, who cares about the utility of the poorest dynasty, that does not invest in education.

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[5] A long and never lasting discussion is documented in the literature on the causes of bequests, which in practice change during the life cycle and are related to many different aspects covered in the literature. Recently, Kopczuk and Lupton (2007) and Kopczuk (2009) document the different bequest motives. Abel (1985) is a pioneer paper analyzing accidental bequests.

[6] Galor and Zeira (1993) show that the imperfection in financial markets, together with the indivisibility of the investment of education, constitute a market failure that causes inequality to be transmitted among generations.

2. Bequests and Investment in Education

2.1 Modeling Investment in Education

In this section I build a model in order to study the behavior of bequests under the presence of investment in education. Bequests are used to finance education of a future generation[7], and thus they are a source of income inequality. Following Galor and Zeira (1993), I will assume three different dynasties - symbolized by sub-index i (i=1,2,3) – that decide whether investing or not in education. Investment in education requires a minimal level X^*(i.e., there is an indivisibility of human capital up to a certain level): investing above this level provides a net return on education in the labor market, in the form of hourly wage, w_S. Otherwise, the hourly wage level is of subsistence. Thus, the net wage under investment in educations is:

(1) $w_{Si} = (1-\tau) n_S X_i, X_i > X^*,$

where τ represents the linear income tax rate and n_S is the gross return on education. The cost of education is given by:[8]

(2) $g(X_i) = \frac{X_i^{1+\lambda_i}}{1+\lambda_i}$, $X_i > X^*$

Where g is the cost of education function, and λ_i is a parameter that represents the cost of education, including financial costs. This function is characterized by a plausible property: higher marginal education costs for high levels of education.

A dynasty that receives a low bequest is required to access the credit market, imposing an additional cost. However, when λ is high, this dynasty may not be able to invest the minimal amount in education X^*. As in Galor and Zeira (1993), I distinguish among three (stationary[9]) dynasties: 1) A poor dynasty (i=1) that

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[7] A practical implementation of the model's steady state would be that parents transfer gifts and inheritances to their sons, who use them for financing education of their own sons (i.e., the effective transfer takes place from grand parents to grand sons).

[8] This function was used by Sheshinski at his graduate Public Economics course, and by Laffont (1994, p.220).

[9] In my model the dynasties act in a stationary steady state. For a discussion about the dynamics see Galor and Zeira (1993).

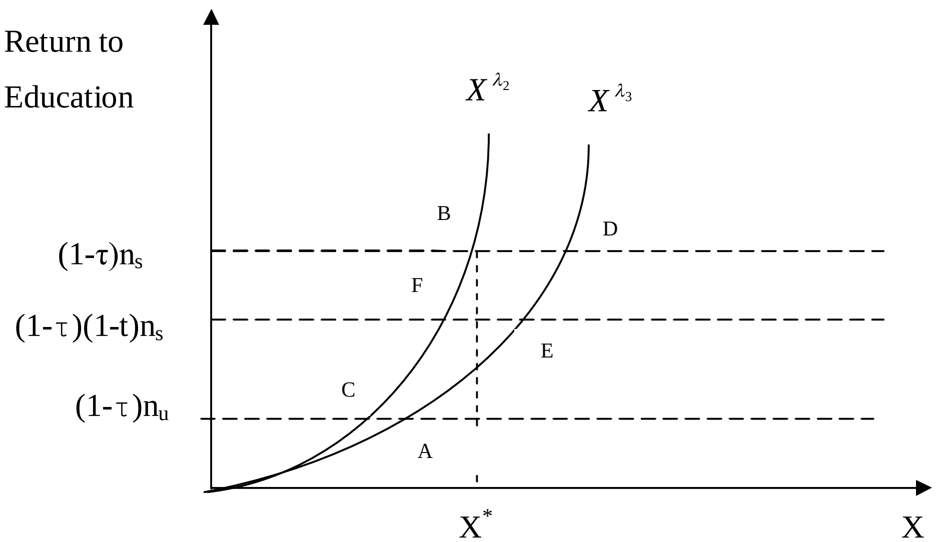
leaves a bequest lower than X^*, that does not invest in education, and consequently its wage is $n_u < n_s$; 2) A middle class dynasty (i=2) with a cost parameter λ_2 that includes financial intermediation costs, and invests X^*; i.e., it is indifferent between investing or not in education; and 3) A dynasty with a high bequest (i=3) that does not borrow, and consequently has a lower parameter, λ_3 – that always invest in education. The F.O.C. for investing in education is:

$$(3)\quad (1-\tau)n_S = X_i^{\lambda_i}, \quad i = 2,3$$

i.e., investment in education differs depending on whether the worker is a lender or a borrower. Note that for i=2 the education cost is higher, since the worker is a borrower. Thus, the second dynasty will have a lower investment in education.

In Figure 1 I use sub-indexes 2 and 3 to identify the two cost parameters ($\lambda_2 > \lambda_3$):

Figure 1: Investment in Education



Let us start by characterizing the poor dynasty. This dynasty receives a low bequest, and consequently does not invest in education. The net wage is $(1-\tau)n_u$, and the relevant point is A, which implies not investing in education. Borrowing will make things even worse for this dynasty, since the marginal cost would become higher (represented by the point C that is left to A). Thus, in both scenarios this dynasty does not invest in education.

Concerning the second ("middle") and third ("rich") dynasties, note that the relevant cost parameters are different. For the middle dynasty the education costs include borrowing costs that are not present for the rich dynasty, which implies that $\lambda_2 > \lambda_3$. Thus, the middle dynasty is at point B and invests X^* which is at the border of indifference concerning investing in education.

The rich dynasty, that receives a high bequest, would be at point D, which implies investing in education. Moreover, note that this dynasty would invest in education even if there is a tax on bequests (f.e., in point E), represented by t, as long as the tax does not reduce the return to education to a desired level of investment that is lower than X^*. Clearly the second dynasty would be affected by the imposition of t, causing it to move to point F, which implies no investment in education.

The main purpose of this paper is to learn about the optimal inheritance tax under different types of central planners, a task that will be performed in the next section. First, I analyze whether the three dynasties will provide bequests under different types of uncertainty. Consistently with the framework presented in Galor and Zeira (1993), in all models I will assume that the minimum level of education investment, X^*, is the one related to the second dynasty under income certainty.

Note that in this model the investment in education of the dynasty i (i=2,3) will be:

(4) $X_i = [(1-\tau)n_s]^{\frac{1}{\lambda_i}}$

and in particular, the minimum level of education, X^*, is:

(5) $X^* = [(1-\tau)n_s]^{\frac{1}{\lambda_2}}$

Consequently, X_1 and X_3 will be 0 and higher than X^*, respectively. This latter result derives from the fact that $\lambda_2 > \lambda_3$, and consequently:

(5)' $X_3 = [(1-\tau)n_s]^{\frac{1}{\lambda_3}} > X^*$

Note that following this inequality, the wage of the third dynasty will be higher than the one of the second dynasty.

2.2 Investment in education and bequests

2.2.1 The dynasty problem

In order to work with the simplest framework I start by introducing a subsequent generations model where individuals live one period and the three dynasties are in a steady state. The best way to think of this model would be to assume that grand parents are willing to transfer resources to grand children (identified by sub-index k) in order to finance their investment in education. I look at the dynasties, and consequently the simultaneous decision is on labor supply and investment on education for the next generation, who – once the decision of investment in education has been taken - supplies one labor unit.[10] The investment in education is facilitated by the bequest. The government redistribute income by collecting an income tax on labor income, and uses tax revenues to finance a demogrant, which for simplicity is delivered to the grand parents generation (identified by sub-index F). As in dynamic programming, I first solve the problem for the kid, and once I obtain the solution I solve the problem for the grand parent. At this stage I assume that there is no inheritance tax (it will be introduced in the next section). The consumption functions for dynasty i that invests in education (i=2,3) are:

$$(6)\quad \begin{aligned} c_{Fi} &= (1-\tau) w_{Fi} l_{Fi} + A - X_i \\ c_{Ki} &= n_S (1-\tau) X_i - g(X_i) \end{aligned}$$

where A and τ are the demogrant and linear income tax, respectively.[11] The dynasty decides about allocation of labor and education. From the point of view of the grand parent, the investment of education is provided according to the optimal level needed for the kid. This decision is then implemented by the bequest trespassed to the kid. Concerning fertility, I follow the findings in the income distribution literature, which imply an asymmetric behavior among poor and rich dynasties:[12] for the poor dynasty (i=1) I assume the existence of N children, while

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[10] This assumption is similar to Galor and Zeira (1993). Introducing disutility of labor for the kid would not affect the main results.

[11] For simplicity I assume that government intervention concentrates on income redistribution. Implicitely, I assume an exogenous level of public goods (not including education).

[12] Dahan and Tsiddon (1998) consider endogenous fertility and find that poor dynasties will have more children. This result is related to the cost of acquiring human capital and to the cost of forgone earnings.

for the middle and rich dynasties (i=2,3), that invest in education, I assume that there is a single kid. For simplicity I have assumed a zero interest rate.

For the dynasty that does not invest in education (i=1) the budget constraints are:

$$(7)\quad \begin{aligned} c_{Fi} &= (1-\tau)n_U l_{Fi} + A \\ c_{Ki} &= n_u(1-\tau) \end{aligned}$$

For the dynasties that invest in education, the first step is to solve its optimal level. From equations 3 and 6 I obtain:

$$(8)\quad \begin{aligned} X_i^* &= [(1-\tau)n_s]^{\frac{1}{\lambda_i}} \\ c_{Ki} &= [(1-\tau)n_s]^{\frac{1+\lambda_i}{\lambda_i}} - \frac{[(1-\tau)n_s]^{\frac{1+\lambda_i}{\lambda_i}}}{1+\lambda_i} = \frac{\lambda_{\mathrm{i}}}{1+\lambda_{\mathrm{i}}}[(1-\tau)n_s]^{\frac{1+\lambda_i}{\lambda_i}}, \ \mathrm{i}=2,3 \end{aligned}$$

The optimization problem from the point of view of a parent in a dynasty that invests in education (i=2,3) is (the bar above c_K means that it is given, according to equation 8):

$$(9)\quad \underset{l_{Fi}}{MAX}\ U_i = \left[\ln[(1-\tau)w_{Si}l_{Fi} + A - \bar{X}_i] + \ln(1-l_{Fi}) + \ln(\bar{c}_{Ki})\right] \qquad \mathrm{i}=2,3$$

Where:

$$(10)\quad w_{si} = [(1-\tau)n_s]^{\frac{1+\lambda_i}{\lambda_i}},\ \mathrm{i}=2,3$$

Applying the F.O.C. for labor derives in the following labor supply:

$$(11)\quad \begin{aligned} \frac{\partial U_i}{\partial l_{Fi}} &= \frac{(1-\tau)w_{Si}}{(1-\tau)w_{Si}l_{Fi}+A-X_i} - \frac{1}{1-l_{Fi}} = 0 \\ l_{Fi} &= \frac{1}{2}\left(1 - \frac{A-\bar{X}_i}{(1-\tau)w_{Si}}\right),\ w_{Si} > \frac{A-X_i}{1-\tau} \end{aligned}$$

i.e., income tax and the demogrant distort labor supply. The tax works through the intensive margin, and the demogrant causes an income effect. The bequest trespassed to children in the form of investment in education, implies a negative income effect which increases labor supply. It is assumed that the normalized hourly wage for a skilled individual is higher than the demogrant (net of inheritance) divided by a net dollar acquired through participation at the labor market. This assumption is equivalent to assuming that the skilled worker participates, which is clearly in line with stylized facts at labor markets.

For the poor dynasty (i=1), that does not leave bequests, the maximization problem from the point of view of the parent is:

$$(12)\quad \underset{l_{F1}}{MAX}\ U_1 = \left[\ln[(1-\tau)n_u l_{F1} + A] + \ln(1-l_{F1}) + N\ln(\bar{c}_K) \right]$$

And the solution is:

$$(13)\quad \begin{aligned} &\frac{\partial U_1}{\partial l_{F1}} = \frac{(1-\tau)n_u}{(1-\tau)n_u l_{F1}+A} - \frac{1}{1-l_{F1}} = 0 \\ &l_F = \frac{1}{2}\left(1-\frac{A}{(1-\tau)n_u}\right),\ n_u \geq \frac{A}{1-\tau} \end{aligned}$$

i.e., since the parent does not invest in education there is no bequest and the kid consumes according to the inelastic unit of labor and the unskilled normalized hourly wage. For simplicity, I will assume that n_u is equal to the threshold wage, and consequently the poor individual is indifferent about participating in the labor market (abolishing this assumption does not change main results).

2.2.2 The dynasty problem with unintended bequests

As in previous literature on bequests, I shall extend the model to a case inwhich individuals leave an "accidental" bequest. I start by solving the problem for a representative agent, and later I refer to the three levels of w. The problem for a grand parent while he is young, who leaves unintended bequests to a single kid because of a positive probability of surviving to the second period, is:

$$(14)\quad Max\left\{ \ln(c_{1i}) + \delta(1-p)\ln[u(c_{2i})] + [\ln(1-l_i)]^{\theta} + \ln(\bar{c}_K) \right\}$$

Where δ represents the subjective discount rate, p (0<p<1) is the probability of demise at the end of the first period, c_1 and c_2 represent consumption in periods 1 and 2 ("young" and "old") respectively, and c_K is the consumption of the single kid. As before the consumption of the kid from the point of view of the grand parent is given. For simplicity I assume that θ=0.

In order to analyze a more general case I will allow for income uncertainty. Among the different cases of income uncertainty analyzed by Strawczynski (1999), the single relevant case in the present model is Second Period Income

Uncertainty (SPIU) which will be introduced through an additive macroeconomic shock, as appears in the following budget constraint (for dynasties 2 and 3):

(15) $c_{2i} = R[n_s(1-\tau)X_i - X_i^* - c_{1i} - a_i] + Aa_i + \varepsilon$

Where n represents first-period hourly wage, a (≥ 0) is the demand for annuities, R is the return to a risk-less bond and ε is the additive macroeconomic shock on old-age income, with a symmetric non-degenerate distribution and mean 0. Basically we shall assume that annuities are actuarially fair; i.e., A=R/(1-p). However, I will also assume that they are not (A<R/(1-p)), creating an incentive for individuals to save on risk-less bonds and thus leaving "accidental bequests". These bequests would occur in the case of demise at the end of the first period.

I recall the fact that from the point of view of the grand parent the solution of the kid is given, according to equation 3.

2.2.3 Income certainty (ε=0)

The consumer decides between allocating savings to risk-less bonds and/or to actuarially fair annuities. Since in the second period the single relevant case from the consumer's point of view is the state of nature in which he is alive, risk-less bonds clearly constitute a dominated asset, since their return equals R, which is lower than the return on actuarially fair annuities, A. Consequently in this case all saving resources are allocated to annuities[13] and the first order condition is:

(16)
$$\frac{1}{c_{1i}} = \frac{R\delta}{[n_s(1-\tau)-1]X_i - c_{1i}}$$
$$c_{1i} = \frac{[n_s(1-\tau)-1]X_i}{1+R\delta}; c_{2i} = \frac{R\delta[n_s(1-\tau)-1]X_i}{1+R\delta}$$

Since annuities are actuarially fair, they provide full insurance against the existence of undesired savings at the end of first period.

2.2.4 SPIU (ε≠0)

Uncertainty is introduced through an additive component, ε, which has a non-degenerate distribution. Note that in this case the introduction of income uncertainty does not alter the fact that risk-less bonds are a dominated asset, since they provide a lower rate of return for transferring resources to the second period.

[13] See Strawczynski (1999).

The existence of SPIU is subject on being alive; thus, the single relevant state of nature is being alive in the second period, and consequently the individual allocates all the savings to annuities. The F.O.C. is:

(17) $$\frac{1}{c_{1i}} = E\left(\frac{R\delta}{[n_s(1-\tau)-1]X_i - c_{1i}}\right)$$

As in the certainty case, actuarially fair annuities provide full insurance for transferring resources to the second period. Note that as a consequence of SPIU individuals will engage in precautionary savings. Thus, the demand for annuities in this case will be higher than in the case of income certainty.

However, in these two cases there would be no accidental bequests. Thus, we turn to a more realistic case, in which actuarially fair annuities are not available and consequently individuals leave accidental bequests.

2.2.5 Bequests under actuarially unfair annuities [no annuities case]

In this case savings will be allocated to risk-less bonds and will result on "accidental bequests" (Abel, 1985).

Income certainty (ε=0)

For an educated individual whose grand parent died at the end of the first period, the minimal bequest out of the net wage equals:

(18) $$b^*_i = [n_s(1-\tau)]^{\frac{1}{\lambda_i}}$$

The kid will use this bequest for achieving the desired level of education. As in Abel (1985), the dynamics of the income distribution is dependent on the number of generations in which grand parents died young. We calculate next the accidental bequests for the three dynasties, which differ on the bequest received.

The accidental bequest of the second dynasty, that is transferred to a single kid, can be calculated using the following equation:

(19) $$b_2 = \sum_{i=0}^{m} \frac{\delta(1-p)}{1+\delta(1-p)}[n_s(1-\tau)X_2 - [n_s(1-\tau)]^{\frac{1}{\lambda_2}}] = \sum_{i=0}^{m} \theta I_2$$

Where m is the number of generations in which the grand parent died young. In the rich dynasty, characterized by a grand parent that died at the end of the first period in m generations, the bequest will be higher, since his/her income is higher because of the higher investment in education. In this dynasty, the accidental bequest equals:

$$(20)\ \ b_3 = \sum_{i=0}^{m} \frac{\delta(1-p)}{1+\delta(1-p)}[n_s(1-\tau)X_3 - [n_s(1-\tau)]^{\frac{1}{\lambda_3}}] = \sum_{i=0}^{m}\theta I_3$$

For both dynasties the total bequest equals:

$$(21)\quad total\ bequest = b^*{}_i + b_i$$

Finally, I analyze the poor dynasty. Since it does not leave an educational bequest, the single type of bequest, which will be provided to N kids, is accidental:

$$(22)\quad \begin{aligned} Nb_1 &= \sum_{i=0}^{m} \frac{\delta(1-p)}{1+\delta(1-p)}[n_u(1-\tau)] = \sum_{i=0}^{m}\theta I_1 \\ b_1 &= \frac{m\theta I_1}{N} < X^* \end{aligned}$$

Note that in principle a high accidental bequest could allow a kid to invest in education. However, the low income of this dynasty combined with the fact that the bequest is divided among N children, implies that it is plausible to assume that the accidental bequest is lower than X^*. This implies that the bequest is consumed by the kid, and it is not trespassed to the following generation in the form of investment in education.

SPIU

In SPIU there is an increase in the demand for accidental bequests and thus for risk-less bonds. Assuming a uniform and symmetric ditrbution, with probability q for a postive shock, the F.O.C. is:

$$(23)\quad \begin{aligned} \frac{1}{c_{1i}} &= q\left(\frac{R\delta(1-p)}{I_i+\varepsilon-c_{1i}}\right) + (1-q)\left(\frac{R\delta(1-p)}{I_i-\varepsilon-c_{1i}}\right) \\ c_{1i}^{SPIU} &< c_{1i}^{CERTAINTY} \\ b_i^{SPIU} &> b_i^{CERTAINTY} \end{aligned}$$

In order to assure that the present framework is still relevant under income uncertainty, the following assumption is needed concerning the first dynasty:

$$b_1^{SPIU} < X^*$$

In appendix 1 I show that for empirically plausible values of the parameters, this assumption is realistic.

Summarizing, accidental bequests are relevant only when annuities are actuarially unfair. High accidental bequests occur when the grand parent died young for many generations, as shown by Abel (1985). I found that in this case both the second and third dynasties increase accidental bequests when income uncertainty is present.

Note that in previous papers the optimal inheritance tax in the presence of accidental bequests is 100 percent.[14] Here there are two types of bequest: intended bequests for investment in education, and unintended bequests that are accidental. Thus, it will be interesting to discuss in Section 3 whether the presence of investment in education modifies the optimal 100 percent inheritance tax result.

3. The optimal Inheritance Tax in the presence of investment in education

3.1 Education and the Inheritance/Gift tax

Before presenting the solution for the optimal schedule, it is important to discuss the relationship between investment in education and the inheritance/gif tax. It is worth to stress that in reality the expenditures for investment in education are rarely related to the inheritance/gift tax. Tuitions and other educational spending by parents for their children at different countries are not considered as gifts, and thus they are not subject to gift tax rules. In this sense, educational expenditure is actually exempted by policy makers, without any relationship to the inheritance tax threshold. It is important to stress, thus, that the threshold calculated in the next section, shall be considered as a benchmark case for normative analysis, and not as a positive explanation for explaining existing rules on gifts/bequests taxation.

Another important appreciation is that in many countries educational expenditure is exempted by different provisions, strengthening the rationale for looking at education as a crucial feature for analyzing a threshold. For example in the U.S.

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[14] Kopczuk (2003) explains that compared to the first best policy (i.e., in the presence of annuities), a 100 hundred percent tax is not optimal since it implies a welfare loss. He shows that with lack of annuities, the inheritance tax can act as an annuitizing device.

any expenditure on education tuition that is paid directly to an educational institution is exempted from the gift tax. Similarly, grand parents deposits for educational funds of grandchildren are exempted from gift taxes.

3.2 The optimal inheritance tax schedule with investment in education

To calculate optimal taxes we introduce a social planner that maximizes social utility of the three dynasties:

$$(24) \quad \underset{t,\tau}{MAX}\ W = \sum_{i=1}^{3}\left[\frac{U_i^{\,1-\beta}}{1-\beta}\right]$$

Where β is the inequality aversion parameter; β=0 corresponds to the utilitarian social planner, and when β tends to infinity the social planner is Rawlsian; t is the inheritance tax.

In the utilitarian benchmark we can find cases in which it would not be desirable to redistribute, and consequently there wouldn't be a case for government intervention through taxes, that distort individual decisions. Thus, the relevant case is when β>0.

The Rawlsian case is particularly interesting: note that the poor generation does not receive a bequest, and consequently one could think that a tax on bequests will be desired by the Rawlsian planner. However, in this case the inheritance tax distorts the decision for investment in education, and consequently it affects the income tax revenues; since tax revenues finance the demogrant, we shall check what is the optimal tax schedule.

For simplicity I assume that n_u is slightly lower than the threshold. Thus, individuals of the poor dynasty do not work, and their single source of income is A.[15] The Rawlsian planner maximizes A:

$$(25) \quad \underset{t,\tau}{MAX}\ A = \frac{\tau n_s l_2 + \tau n_s l_3 + tX_2 + tX_3}{3}$$

Where, by assumption, $\tau n_S l_2 > tX_2$. This assumption is empirically plausible and related to the fact that labor income is obtained during most years of lifetime.

In the rest of the paper I will concentrate in the case of income certainty. Let us start with the optimal inheritance tax. In the presence of investment in education,

[15] Abolishing this assumption would change the results for the optimal income tax schedule, but would not affect my conclusions on the optimal inheritance tax.

it is optimal to impose a threshold, since it will support investment in education of the second dynasty. I will analyze this point in section 3.3. As in the previous section, I will start by using a standard model of subsequent generations (as in section 2.2.1), and then I turn to a case that adds accidental bequests.

3.3 The optimality of a threshold in the inheritance tax schedule

The following lemma holds:

<u>Lemma 1</u>

The optimal Rawlsian inheritance tax schedule in the presence of investment in education includes a threshold under which the inheritances should not be taxed. This is true also for individuals that leave accidental bequests.

<u>Proof</u>

The relevant equations are: 2 and 25. Adding an inheritance tax would imply that the second dynasty would not invest in education:

(26) $(1-\tau)(1-t)n_S < X_i^{\lambda_2}$

Thus, the income of the second dynasty will be based on n_u, that is lower than n_s. Consequently, according to equation 25 there would be a reduction in A, and the Rawlsian planner would prefer imposing a threshold, under which inheritances/gifts are not taxed.

3.4 The optimal inheritance and income tax rates

Concerning the optimal inheritance tax rate, we shall remember that the second dynasty is indifferent between investing or not in education, and consequently will leave an inheritance of the minimum side (equal to the threshold). Thus, the question of optimality of the inheritance tax depends on the revenue obtained from the third dynasty:

(27) $T(t) = t.[n_s(1-t)(1-\tau)]^{\frac{1}{\lambda_3}}$

The Rawlsian planner will maximize this revenue:

(28) $\underset{t}{MAX}\ T(t)$

And the F.O.C. is:

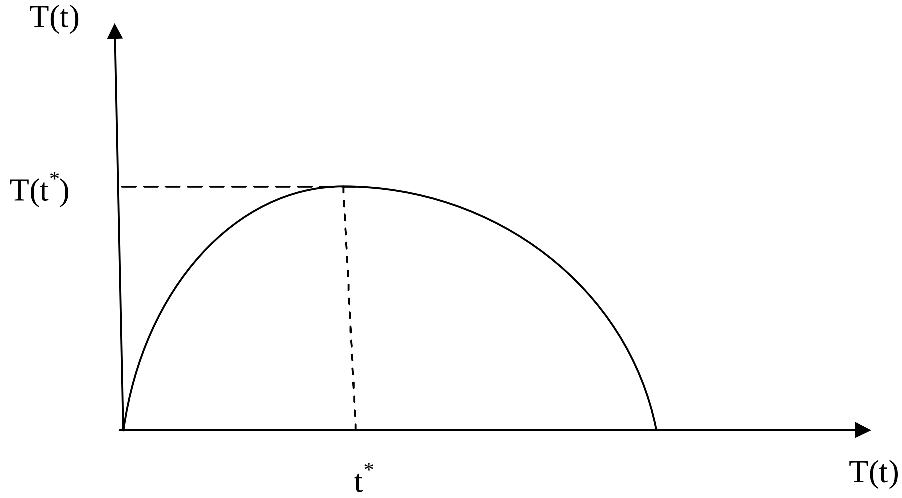
$$[n_s(1-t)(1-\tau)]^{\frac{1}{\lambda_3}} = \frac{t}{\lambda_3} n_s(1-\tau)[n_s(1-\tau)(1-t)]^{\frac{1}{\lambda_3}-1}$$

(29)

$$t^* = \frac{\lambda_3}{1+\lambda_3}$$

Thus, the optimal inheritance tax depends on the cost parameter for the rich dynasty, λ_3. For example, if it equals 1 the optimal tax would be 50 percent. Note that the higher is λ_3, the higher is the tax rate. The intuition for this result is: as λ_3 increases, the demand for education becomes more rigid. Thus, imposing a tax has a smaller deadweight loss, allowing for a higher tax rate. Note that the optimal tax rate represents a maximum at the Laffer curve (Figure 2).

Figure 2: The optimal Inheritance Tax



In Table 3 I present a simulation of the costs parameters, under the assumption that the inheritance of the rich dynasty is, respectively, 3, 5 and 10 times the one of the middle dynasty. The results of the simulations are discussed in section 3.6.

We now turn to the optimal income tax rate. In the linear case the revenue from the income tax equals:

$$(30) \quad T(\tau) = \frac{\tau}{2} n_s \left[\left(1 - \frac{A - X_2}{(1-\tau)n_s} \right) + \left(1 - \frac{A - X_3}{(1-\tau)n_s} \right) \right]$$

Since the Laffer curve for each dynasty differs as a function of X, it is optimal to set a pecewise linear system with two tax rates:

$$(31)\quad \begin{aligned} T(\tau_2) &= \frac{\tau_2}{2} n_s \left(1 - \frac{A - X_2}{(1-\tau_2)n_s}\right) \\ T(\tau_3) &= \frac{\tau_3}{2} \left(1 - \frac{A - X_3}{(1-\tau_3)n_s}\right) \end{aligned}$$

Optimal Rawlsian taxes are obtained by deriving the revenues according to the each one of the tax rates of equation 31, and equalizing to zero. It is easy to show that the optimal tax rate is obtained through the following equation:

$$(32)\quad \begin{aligned} & a\tau_i^2 - 2b\tau_i + b = 0 \\ & \textit{where}: \\ & a = n_s \\ & b = n_s - A + X_i \\ & i = 2,3 \end{aligned}$$

The single feasible solution of this equation is:

$$(33)\quad \tau_i = \frac{b - \sqrt{b}\sqrt{b-a}}{a}$$

Note that since $X_3 > X_2$, it can be shown that $\tau_2 > \tau_3$, i.e., the second tax rate is lower than the first one.[16] In the next sections I present and discuss simulations for calibrating the optimal income tax rates under different assumptions.

3.5 Optimal inheritance tax in the presence of investment in education and accidental bequests

The relevant model is the one presented above in section 2. The kids invest in education and they receive an accidental bequest. Thus, the optimal tax can be obtained by analyzing the decision of investment in education.

For the second dynasty the relevant equations are 21 and 25. According to equation 21, imposing an inheritance tax from the first dollar would imply that this dynasty does not invest in education, and thus it would be sub-optimal from the point of view of a social planner. Equation 25 shows that the additional bequests beyond b^*_2 should be taxed at 100 percent. However, acting this way

[16] In the literaure there is not consensus on the optimal piecewise linear income tax schedule: while Slemrod,Yitzhaki, Mayshar and Ludholm (1994) obtained a similar result, Strawczynski (1998) and Apps, Van Long and Rees (2011) obtained a result according to which the second rate is higher than the first.

would imply hurting the educational bequests of the third dynasty, and consequently in order to set the optimal tax we shall first analyze the bequests provided by this dynasty.

For the third dynasty the relevant investment is given in equation 26, after introducing the inheritance tax. Similarly to the previous sub-section, the revenue of the Rawlsian planner is:

$$(34)\qquad T(t) = t.[n_s(1-t)(1-\tau)]^{\frac{1}{\lambda_3}}$$

Thus the optimal inheritance tax rate for bequests intended to finance investment in education is:

$$(35)\qquad t^{E*} = \frac{\lambda_3}{1+\lambda_3}$$

The question now is whether the revenue from bequests provided by the third dynasty is higher than the loss of revenue implied by not taxing second dynasty accidental bequests at 100 percent.

To check this question[17], and using equations 5', 19 and 35, I compare:

$$(36)\qquad \begin{aligned} &\frac{\lambda_3}{1+\lambda_3}\left[n_s(1-\tau)\right]^{\lambda_3} > \frac{1}{1+\lambda_3}\sum_{i=0}^{m}\theta I_2 \\ &\frac{\lambda_3}{1+\lambda_3}\left[n_s(1-\tau)\right]^{\lambda_3} > \frac{1}{1+\lambda_3} m\theta\left[n_s(1-\tau)\right]^{\frac{1}{\lambda_2}}[n_s(1-\tau)-1] \\ &1+\frac{\lambda_3}{1+\lambda_3}\frac{n_s(1-\tau)^{\lambda_3}}{B} > m\theta n_s(1-\tau), \quad \text{where } B = \frac{1}{1+\lambda_3}n_s(1-\tau)^{\frac{1}{\lambda_2}} \\ &K > H \end{aligned}$$

In Table 3 I use realistic parameters to check this inequality, and I find that for the relevant range of parameters this inequality clearly holds. Thus, for the range in which educational bequests of the third dynasty overlaps with

[17] Assuming no initial wealth.

accidental bequests of the second dynasty, we conclude that the loss of revenue of imposing a 100 hundred tax rate is higher than the benefit of imposing t^{E*}. Thus, a Rawlsian planner will choose t^{E*}.

In summary, the optimal tax schedule is:

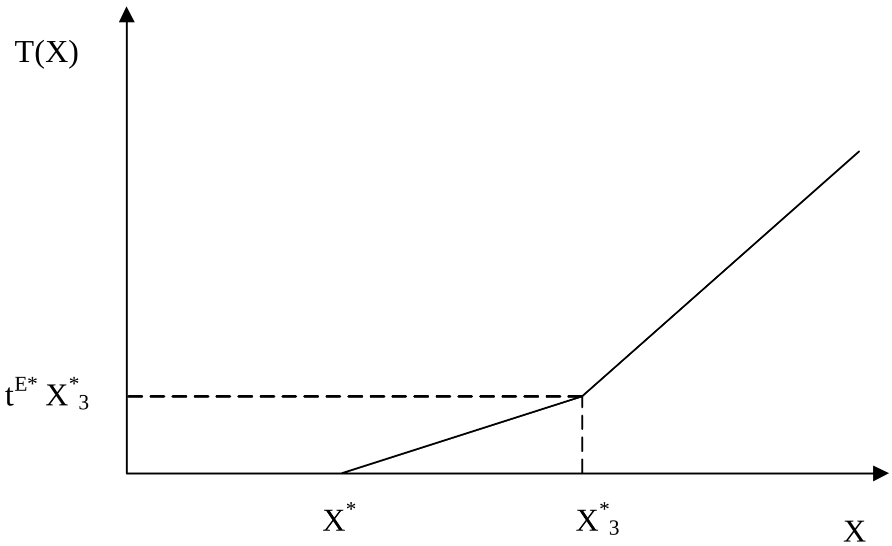
$$\text{for } X < X^*, \quad t = 0$$

$$\text{(37)} \qquad \text{for } X^* < X < X_3^*, \quad t^{E*} = \frac{\lambda_3}{1+\lambda_3}$$

$$\text{for } X > X_3^*, \quad t = 1$$

Figure 3 shows a graphical ilustration of the optimal inheritance tax schedule.

Figure 3: The optimal inheritance tax schedule



This result is very remarkable, since it is opposite to the traditional result of a 100 percent tax rate on accidental bequests. In the presence of investment in education, the optimal Rawlsian tax rate is, for a vast range of accidental inheritances, lower than 100 percent. In the next sub-section I perform simulations in order to assess the optimal tax rates.

3.6 Numerical simulations

In Table 3 I show the results of simulations for three cases, varying according the ratio of the levels of inheritance of the rich dynasty relatively to the middle one (for values of 3, 5 and 10). The simulations are based on different values for the cost parameter of the middle dynasty. In all simulations we assume that the value of θ (which is based on the probability of survival to the second period and the intertemporal discount rate) is one third and that the average number of generations with premature demise is 2. Simulations results show that the inequality shown in equation 36 holds in almost all cases. In any case, I will consider only these cases as the relevant ones for the analysis.

As shown in lemma 1, in all cases there is a threshold under which inheritances are not taxed. According to the simulations, the range of the optimal inheritance tax is between 20 and 42 percent. In order to discuss the most plausible level of the inheritance tax, I shall analyze which of the scenarios included in Table 3 is the most relevant. Piketty (2011) reports that in the US the top 10% owns 72 % of U.S. aggregate wealth, and the middle 40 % owns 26%. This would roughly correspond to the first scenario, in which the rich dynasty inheritances are three times higher than those of the middle dynasty. This means that the optimal range according to my simulations is between 28 and 42 percent.

Simulations show that the higher is the ratio between rich and middle dynasties' bequests, the lower is the inheritance tax but the higher are the piecewise income tax rates. The intuition for this result is that as educational bequests become higher, the higher is the labor supply of grand parents for providing them, allowing for higher income tax rates.

3.7Assessing the desired size of the threshold

In this section I produce an estimate of the desired size of the threshold. In particular, it is interesting to have a benchmark so as to compare with the results shown in Table 2, according to which the average threshold in both developed and developing countries is 16 times the national income per-capita. Note, however, that this high average is related to a small number of extreme cases (Greece, Italy and the U.S. in 2012, among developed economies, and Bulgaria, South Africa and Zimbawe among developing ones).

According to lemma 1, the optimal threshold shall cover the minimal expenditure in education, so as to allow the middle dynasty to invest in education. One way to calculate this level would be to estimate the costs parameters and then use equation 5. Another possible way, which is the one pursued here, is to calculate the necessary expenditure for obtaining a basic education.[18] While the main argument presented in this paper is relevant for primary and secondary education, the inclusion of tertiary education for calculating the minimum level of education shall be discussed. Clearly the failure of achieving financial sources is less relevant for this kind of education, given that students maybe able to finance education by their own, through the financial system, without any dependence on transfers. Following this caveat, I will perform two separate calculations for minimal education: the first one will include pre-school, primary and secondary eduaction, while the second one will additionally include tertiary education.

In order to simulate the optimal threshold I look at the weighted average of expenditure in the different types of education, E, using the following formula:

$$(38) \quad E_j = \frac{\sum_i (\text{Annual education expenditure per student})_i (\text{number of years})_i}{\textit{GDP Per Capita}}$$

Where j=1,2 simbolizes the two measures mentioned above and i represents the different levels of education; note that i=1,2,3 (pre-school, primary and secondary education) under the first measure, while i=1,2,3,4 (adding tertiary eduaction) under the second measure. I use data from the OECD as reported in Table B1.4 at *Education at a glance*, which shows education expenditure at the national level (i.e., including both public and private expenditure). I apply this formula for a list of developed and developing countries. Results are shown the results in Table 4. The average ratio according to the different measures is between 2.5 and 5.5. There are two countries in which the actual ratio is similar to the basic calculation: Netherland and Chile. In all other

[18] Note that the model assumes that education is a private good, while in reallity most countries have a public education system. The discussion on the optimal provision of education is beyond the scope of the present paper.

countries the threshold is either lower or higher than the one implied by the simulations. Note also that the average threshold is lower compared to the average one implemented by countries (Table 2).

4. Summary and Conclusions

This paper calculates optimal inheritance taxes when bequests are intended for financing education. I found that the optimal inheritance tax schedule is progressive, and it includes a threshold – which derives from the indivisibilty of education as an input of production. The threshold is estimated in a range between 2.5 and 5.5 times the gdp per capita. Note, however, that this calculation should be judged only as a normative tool for analyzing the desired exemption for investment in education, which in my framework is useful for analyzing the extent of the threshold within the inheritance tax schedule. A positive analysis, and a better aproximation to the real life threshold, shall inquire into the fixed costs of evaluating inheritances, which may imply a lack of viability for taxing low scale inheritances. A rich description of these issues is provided by Kopczuk (2012, p. 45-46).

Opposite to previous models based on individuals that leave unintended bequests, which shall be taxed at a 100 hundred percent tax rate, in the present model this optimal tax rate is relevant only for high inheritances. For a wide range of accidental inheritances, I found a lower optimal inheritance tax rate. Using empirically plausible values, simulations show that the the optimal inheritance tax ranges between 28 and 42 percent. This range is in line with the policy pursued by many advanced countries implementing the inheritance tax.

Table 1 – Inheritance Tax implementation compared to other taxes

(80 countries surveyed; Source: Income Tax around the World)

	Number of Countries	**Percent**
Countries with income tax	80	100
Developed	23	100
Developing	57	100
Countries with corporate tax	80	100
Developed	23	100
Developing	57	100
Countries with inheritance tax	42	52.5
Developed	13	56.5
Developing	29	50.9

Table 2 – The Characteristics of the Inheritance tax[1]

(Source: Ernst and Young)

Country	Inheritance /Estate Tax (percent)	Tax Threshold[2]	Threshold as % of GDP per capita[3]	Comments
Advanced Economies				
Belgium	3-7			Varies depending on the region of residence.
Czech Republic	0.5-2.5*	-	-	
Denmark	15	47,000	78.4%	
Finland	7-13*	26,500	53.7%	
France	5-45*	10,500	23.8%	
Germany	7-30*	654,350	1,500%	Spouse or common-law spouse of transferor.
Greece	10	788,000	2,910%	
Iceland	10	12,150	28%	
Ireland	25	436,000	918%	Capital Acquisitions Tax (CAT) includes both gift and inheritance tax.
Italy	4	1,327,000	3,659%	
Japan	10-50*	859,000	1,870%	For a single heir, a basic exemption of ¥50 million, plus ¥10 million multiplied by the number of statutory heirs, is deductible from taxable properties.
Luxembourg	0-5	13,000	11.5%	
Netherlands	10-20	157,500	313%	
Norway	6-10*	82,000	84%	

Portugal	10	-		Except for the spouse, ascendants and descendants who benefit from an exemption.
Spain	7.65-34*			Estate and gift tax rates vary depending on the autonomous region.
Switzerland	0-50			No inheritance or gift taxes are imposed at the federal level. Almost all cantons levy separate inheritance and gift taxes. Rates vary widely depending on the canton where the deceased or donor is domiciled.
U.K.	40	527,000	1,365%	
U.S.	18-35*	5 million	10,333%	Changes starting in 2013. See note below.
Developed Average (Marginal)	**16.2 (22)**	**710,000**	**1,653%**	
		Developing Economies		
Angola	10-30	-		
Aruba	2-6	-		
Botswana	5-25*	8,142	86%	Estate income tax.
Brazil	4-8	-		States may levy estate and gift tax on transfers of real estate by donation and inheritance at any rate, up to 8%. A rate of 4% generally applies in Rio de Janeiro and Sao Paulo.

Brunei Darussalam	3	1,607,000	4,400%	
Bulgaria	0.4-0.8	945,000	13,122%	
Chile	1-25*	41,500	290%	Estate and gift tax is a unified tax.
Colombia	20	17,160	240%	
Croatia	5	8,800	60%	
Dominican Republic	1	132,000	2,340%	
Ecuador	5-35*	58,680	1,326%	
Equatorial Guinea	10	200	1.4%	
Georgia	20	620	20%	Inheritances and gifts are subject to general income taxation.
Guatemala	0-14	-		Personal.
Jamaica	1.5	1,160	21%	Transfer tax is payable at the following rates on the transfer of land and shares in a Jamaican company.
Korea	10-50*	531,600	2,334%	For a spouse, For child 27,000$ - 117%.
Lebanon	3-12*	20,000	202%	
Lithuania	5-10	-		Close relatives, such as children, parents, spouses and certain other individuals, may be exempt from this tax.
Macedonia	0-5			Inheritances and gifts are subject to tax if the market value of the inheritance or gift is higher than the amount of the average annual salary in the preceding year.

Malawi	5-11*	180	51%	Estate duty
Philippines	5-20*	4,740	213%	
Poland	3-20			Under specific conditions, the closest relatives of the donor or the deceased are exempt.
Puerto Rico	10	10,000	41%	
Senegal	3-50*	150	14%	
Serbia	2-2.5			Depending on the value of the tax base.
Singapore	2-20*	16,000	32.5%	Resumed in February 2011.
Saint Maarten	2-6	-		
Slovenia	5-30	6,600	27%	Spouses, children and their spouses, and stepchildren are not subject to the tax.
South Africa	10-40	453,000	5,616%	
Taiwan	10	360,000	1,791%	
Turkey	1-10*	97,000	922%	
Venezuela	1-25	-		
Zimbabwe	5	50,000	3,537%	
Developing Average (Marginal)	**10.9 (16.4)**	**189,980**	**1,595**	
Total Avg.	**12.8**	**386,744**	**1,617**	

[1]For first degree relatives inheritance.

[2]Valued in U.S. dollars.

[3]GDP per capita in 2011 from the IMF - World Economic Outlook.

*Rated tax, higher tax represents the highest level of tax. Since 2013 the threshold and maximum rate in the U.S. will be 1 million and 55%, respectively.

Table 3: Numerical Simulation*

Case 1: Rich over middle dynasty bequest equals to 3

Variable	$\lambda_2 = 4$	$\lambda_2 = 2$	$\lambda_2 = 1$	$\lambda_2 = 0.8$
λ_3	0.73	0.616	0.475	0.396
t	0.42	0.38	0.32	0.28
K	29.1	5.3	2.3	1.95
H	1.5	1.7	1.8	1.92
τ_1	0.05	0.064	0.102	0.18
τ_2	0.051	0.063	0.103	0.19

Case 2: Rich over middle dynasty bequest equals to 5

Variable	$\lambda_2 = 4$	$\lambda_2 = 2$	$\lambda_2 = 1$	$\lambda_2 = 0.8$
λ_3	0.515	0.45	0.395	0.319
t	0.34	0.31	0.28	0.24
K	19.3	4.7	2.3	1.9
H	1.8	1.8	1.9	2
τ_1	0.102	0.115	0.17	0.256
τ_2	0.104	0.118	0.24	0.274

Case 3: Rich over middle dynasty bequest equals to 10

Variable	$\lambda_2 = 4$	$\lambda_2 = 2$	$\lambda_2 = 1$	$\lambda_2 = 0.8$
λ_3	0.515	0.45	0.395	0.319
t	0.27	0.25	0.24	0.2
K	15.3	4.3	2.2	1.9
H	1.9	2	2	2
τ_1	0.16	0.21	0.37	0.49
τ_2	0.18	0.23	0.47	0.68

* Simulations show the optimal inheritance tax on educational bequests. As shown in section 3.5, beyond rich dynasty's educational bequests the optimal tax is 100%.

Table 4: The Optimal threshold

Country	E_1	E_2
Belgium	3.3	4.5
Brazil	2.5	5.5
Chile	2.6	4
Czeck Republic	2.7	3.7
Denmark	3.6	4.9
Finland	2.8	4
France	3.4	4.6
Germany	2.9	4
Hungary	3.5	4.6
Iceland	3.7	4.5
Italy	3.7	4.5
Japan	3.7	4.5
Mexico	3.3	3.8
Netherland	3.3	4.2
Norway	2.8	3.7
Poland	3.7	4.7
Portugal	3.8	5.2
Spain	3.5	4.7
Switzerland	3.7	5.2
UK	3.6	4.9
US	3.4	5.1
Average	3.2	4.5

Appendix 1 – Accidental bequests of the poor dynasty

In this appendix I use plausible empirical values for the parameters, so as to check whether accidental bequests of the poor dynasty are lower than the minimum level of education.

For this purpose I assume that the ratio of skilled and unskilled wage equals 3; income uncertainty is symmetric with q=1-q=0.5; the income shock, □, equals 0.3; and finally, N equals 3 (which is conservative for poor dynasties).

Concerning the additional parameters, I use the same values as used for simulations with skilled individuals. By applying equation 23, I get the folowing results:

$$c_{1i}^{SPIU} = 0.54 < c_{1i}^{CERTAINTY} = 0.67$$

$$\frac{b_i^{SPIU}}{N} = 0.154 > \frac{b_i^{CERTAINTY}}{N} = 0.11$$

These numbers must be compared to the minimum level of education, X^*, which appears in the following table:

	λ_2 = 4	**λ_2 = 2**	**λ_2 = 1**	**λ_2 = 0.8**
X^*	3.95	3	1.73	1.32

In summary, these results confirm that even in the case of accidental bequests under SPIU, the accidental bequests of the poor dynasty are too low, and thus they do not allow investing in education.

References

Abel, Andrew (1985), "Precautionary savings and accidental bequests", *American Economic Review*, vo. 75 no 4, 777-791.

Apps, Patricia, Ngo Van Long and Ray Rees (2011), "Optimal piecewise linear income taxation", CEPR Discussion Paper no 655.

Blumkin, Tomer and Efraim Sadka (2003), "Estate taxation with intended and unintended bequests", *Journal of Public Economics* 88, 1-21.

Dahan, Momi and Daniel Tsiddon (1998), "Demographic transition, income distribution and economic growth", *Journal of Economic Growth*, 3, 29-52.

Farhi, Emmanuel and Iv?n Werning (2010), "Progressive estate taxation", *Quarterly Journal of Economics*, may, 635-673.

Gales William, James R. Hines Jr. and Joel Slemrod (2001), *Rethinking estate and gift taxation*, The Brookings Institution.

Graetz, Michael and Ian Shapiro (2005), *Death by a thousand cuts: the fight over taxing inherited wealth*, Princeton, NJ: Princeton University Press.

Galor, Oded and Joseph Zeira (2003), "Income distribution and macroeconomics", *Journal of Economic Studies*, Vol. 60 No 1, 35-52.

Kopczuk, Wojciech (2003), "The trick is to live: is the estate tax social security for the rich?", *Journal of Political Economy* 111 (6), 1318-1341.

Kopczuk, Wojciech and J. Lupton (2007), "To leave or not to leave: the distribution of bequest motives", *Review of Economic Studies* 74(1), 207-235.

Kopczuk, Wojciech (2009), "Economics of estate taxation: review of theory and evidence", *Tax Law Review* 63(1), 139-157.

Kopczuk, Wojciech (2012), "Taxation of intergenerational transfers and wealth", draft in preparation for Volume 5 of the *Handbook of Public Economics*.

Laffont, Jean-Jacques (1994), *Fundmentals of Public Economics*, The MIT Press, Cambridge, Massacusetts, 4th Edition.

Slemrod Joel, Shlomo Yitzhaki, Joram Mayshar and Michael Lundholm (1994), "The optimal two-bracket linear income tax', *Journal of Public Economics* 53 (2), 269-290.

Piketty, Thomas (2011), "On the long-run evolution of inheritance: France 1820-2050", *The Quartely Journal of Economics*, vol. CXXVI (3), pp. 1071-1131.

Piketty, Thomas and Emmanuel Saez (2012), "A theory of optimal capital taxation", CEPR Discussion Paper no 8946.

Strawczynski, Michel (1993), "Income uncertainty, bequests and annuities", *Economics Letters* 42 (2-3), 155-158.

Strawczynski, Michel (1998), "Social insurance and the optimal piecewise linear income tax", *Journal of Public Economics* 69 (3), 371-388.